Imaginary numbers and quadratic equations

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Using the imaginary number i it is possible to solve all quadratic equations.

Example Use the formula for solving a quadratic equation to solve $x^2 - 2x + 10 = 0$.

Solution We use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

With $a = 1$, $b = -2$ and $c = 10$ we find

$$\begin{aligned} x &= \frac{2 \pm \sqrt{(-2)^2 - (4)(1)(10)}}{2} \\ &= \frac{2 \pm \sqrt{4 - 40}}{2} \\ &= \frac{2 \pm \sqrt{-36}}{2} \\ &= \frac{2 \pm 6\mathrm{i}}{2} \\ &= 1 \pm 3\mathrm{i} \end{aligned}$$

There are two solutions: $x = 1 + 3\mathrm{i}$ and $x = 1 - 3\mathrm{i}$.

Example Use the formula for solving a quadratic equation to solve $2x^2 + x + 1 = 0$.

Solution We use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

With $a = 2$, $b = 1$ and $c = 1$ we find

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1^2 - (4)(2)(1)}}{2(2)} \\ &= \frac{-1 \pm \sqrt{-7}}{4} \\ &= \frac{-1 \pm \sqrt{7}\,\mathrm{i}}{4} \\ &= -\frac{1}{4} \pm \frac{\sqrt{7}}{4}\mathrm{i} \end{aligned}$$

There are two solutions: $x = -\frac{1}{4} + \frac{\sqrt{7}}{4}\mathrm{i}$ and $x = -\frac{1}{4} - \frac{\sqrt{7}}{4}\mathrm{i}$

We have seen how we can write down the solution of any quadratic equation.

A number like $x = -\frac{1}{4} + \frac{\sqrt{7}}{4}\mathrm{i}$, which has a real part, (here the real part is $-\frac{1}{4}$), and an imaginary part, (here the imaginary part is $\frac{\sqrt{7}}{4}$), is called a **complex number**. We will describe complex numbers more formally in the next unit.

